

# Multi-Client Attribute-Based and Predicate Encryption, Revisited

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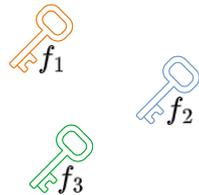


# Attribute-Based Encryption (ABE) [SW05]

$\text{Enc}(\text{mpk}, x, \mu) \rightarrow \text{ct}_x$

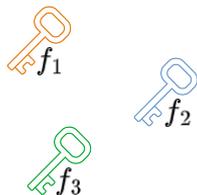


$\text{KeyGen}(\text{msk}, f) \rightarrow \text{dk}_f$



# Attribute-Based Encryption (ABE) [SW05]

$\text{Enc}(\text{mpk}, x, \mu) \rightarrow \text{ct}_x$



$\text{KeyGen}(\text{msk}, f) \rightarrow \text{dk}_f$

$\text{Dec}(\text{dk}_{f_i}, \text{ct}_x)$



if some  $f_i(x) = 1$



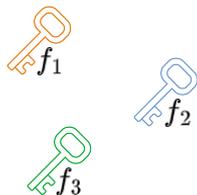
if all  $f_i(x) = 0$

# Attribute-Based Encryption (ABE) [SW05]

attribute-based encryption: **public** input  
predicate encryption: **private** input

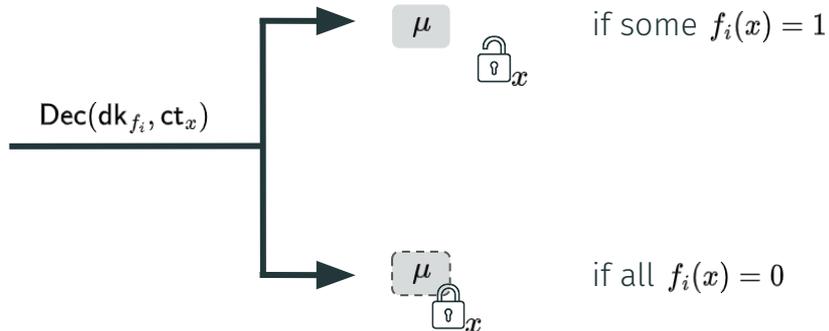
↓

$$\text{Enc}(\text{mpk}, x, \mu) \rightarrow \text{ct}_x$$

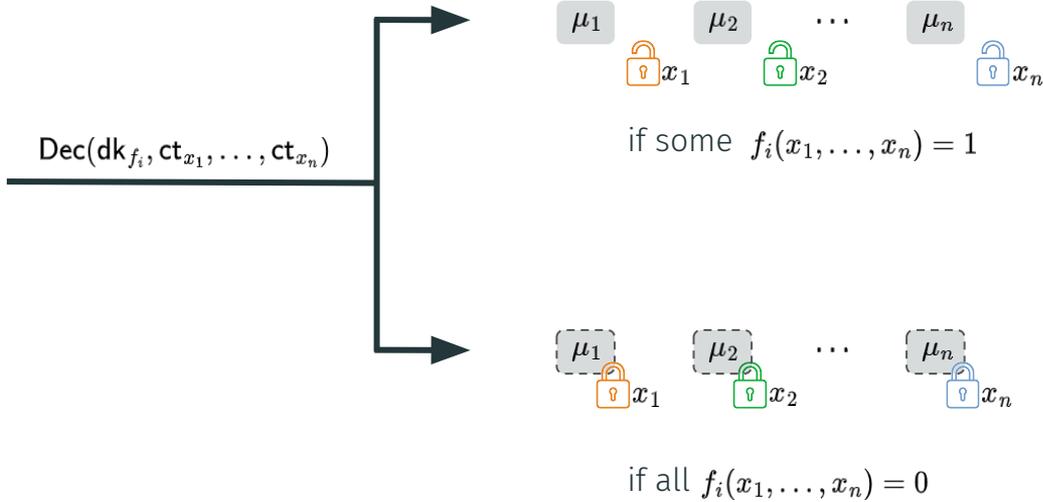
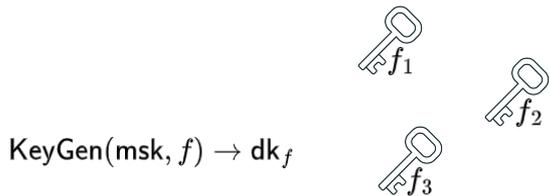
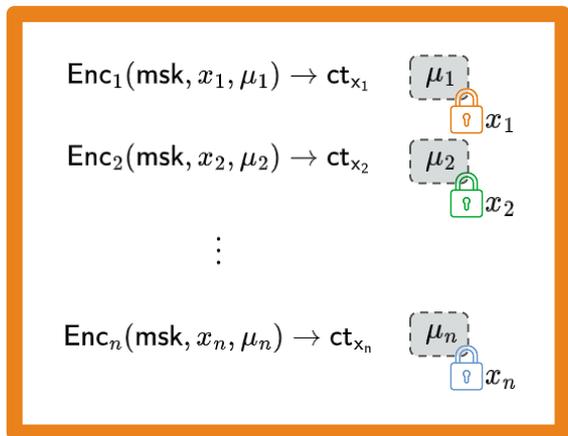


$$\text{KeyGen}(\text{msk}, f) \rightarrow \text{dk}_f$$

↑  
public input

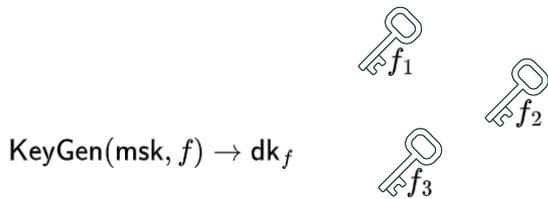
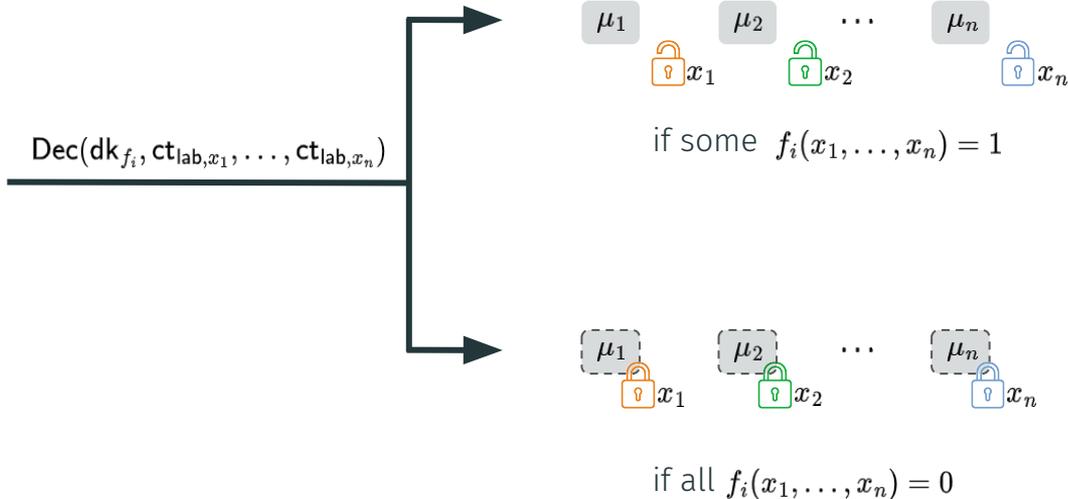
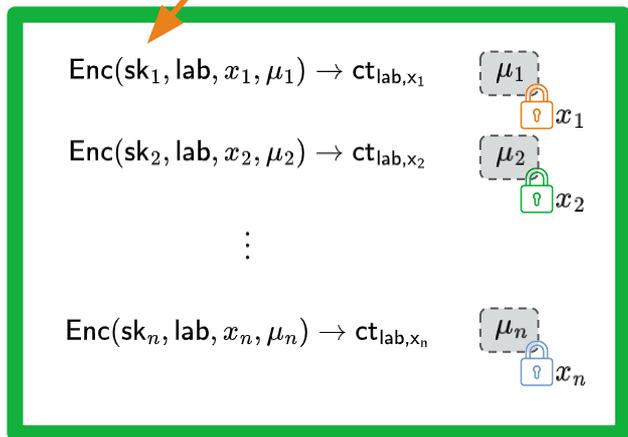


# Multi-Input Attribute-Based Encryption (MI-ABE) [BJK<sup>+</sup>18]



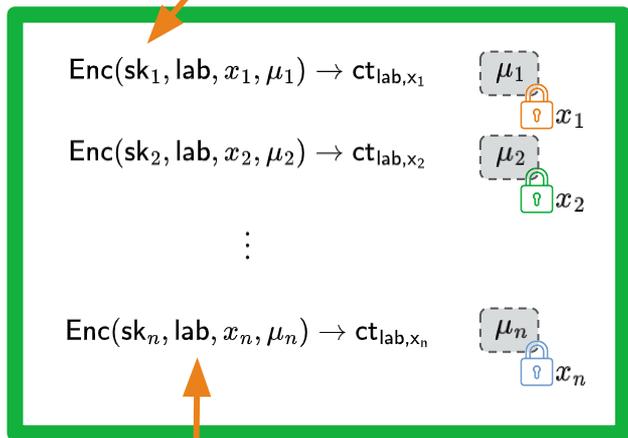
# Multi-Client Attribute-Based Encryption (MC-ABE)

1st new feature:  
separation & corruption of secret keys

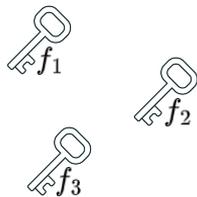


# Multi-Client Attribute-Based Encryption (MC-ABE)

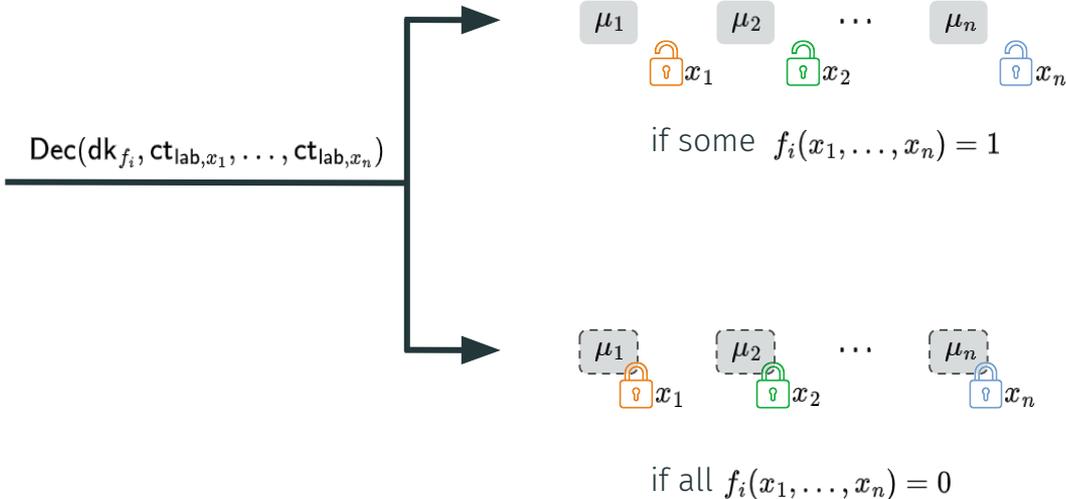
1st new feature:  
separation & corruption of secret keys



2nd new feature:  
encryption w.r.t. labels

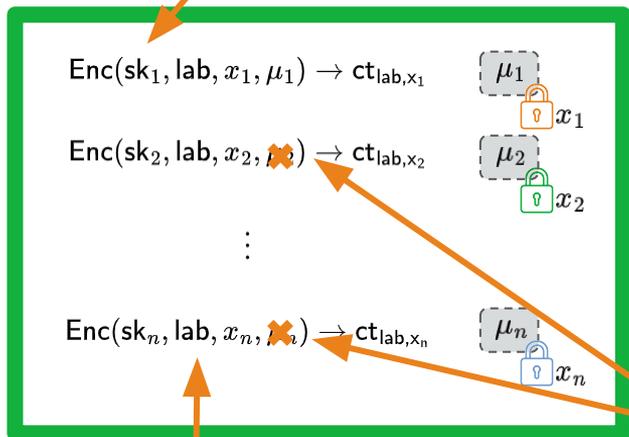


$$\text{KeyGen}(\text{msk}, f) \rightarrow \text{dk}_f$$



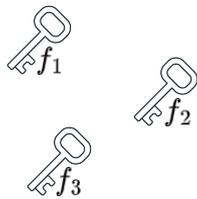
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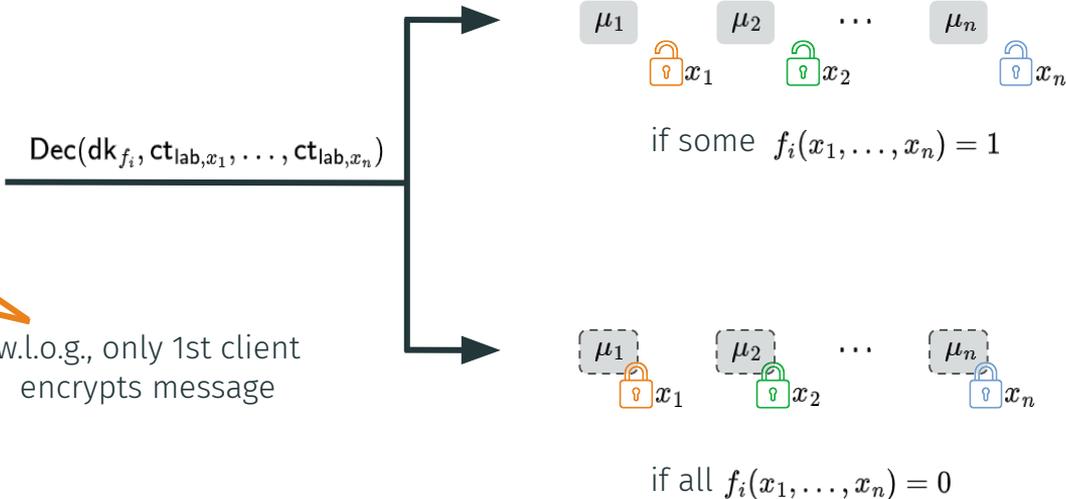


2nd new feature:  
encryption w.r.t. labels

$$\text{KeyGen}(\text{msk}, f) \rightarrow \text{dk}_f$$



w.l.o.g., only 1st client  
encrypts message



# MI-ABE and MC-ABE for Conjunction Policies

↑  
 $f(x_1, \dots, x_n) = f_1(x_1) \wedge \dots \wedge f_n(x_n)$

Work	Policy Class	Assumption	Corruption	Labels	Collusions
[C:ATY23]	NC <sup>1</sup>	MDDH	✓	✗	✓
[this work]	NC <sup>1</sup>	MDDH	✓	✓	✓
[EC:FFMV23]	P	LWE	✓	✗	✗
[this work]	P	LWE	✓	✓	(✓)

# MI-ABE and MC-ABE for Conjunctive Policies

$$f(x_1, \dots, x_n) = f_1(x_1)$$

Generic Compiler of [TCC:PS24]

- MC-ABE  $\rightarrow$  MC-PE
- constant arity
- based on LWE

Work	Policy Class	Assumption	Corruption	Labels	Collusions	Attribute-Hiding
[C:ATY23]	NC <sup>1</sup>	MDDH	✓	✗	✓	✓
[this work]	NC <sup>1</sup>	MDDH	✓	✓	✓	✓
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[this work]	P	LWE	✓	✓	(✓)	✓

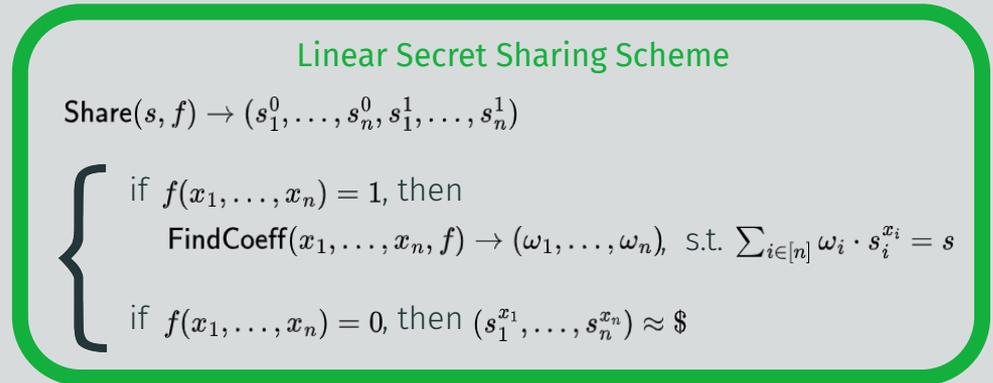
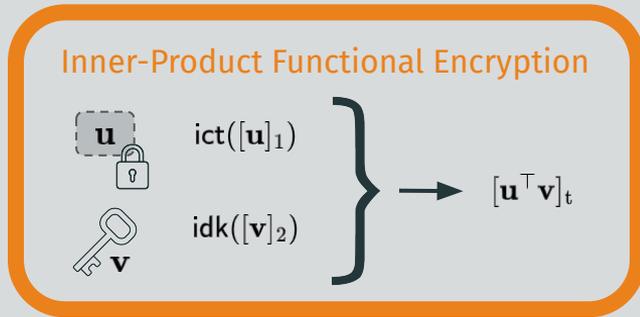
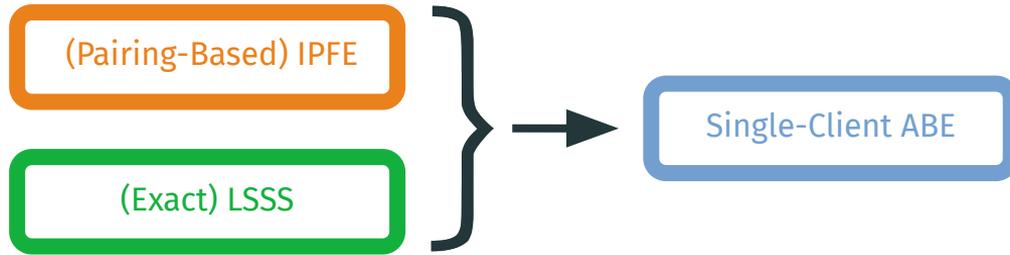
# MI-ABE and MC-ABE for Global Policies

Work	Policy Class	Assumption	Corruption	Labels	Arity	Attribute-Hiding
[C:AYY22]	NC <sup>1</sup>	KOALA, LWE	✗	✗	2	✓
[C:ARYY23]	P	private-coin evasive LWE, tensor LWE, LWE	✗	✗	poly	✗
[TCC:PS24]	NC <sup>0</sup> (and more)	SXDH	✓	✓	poly	✗
[this work]	succinctly enumerable	MDDH	✓	✓	poly	✗
[this work]	succinctly enumerable	public-coin evasive LWE, LWE	✓	✓	poly	✗

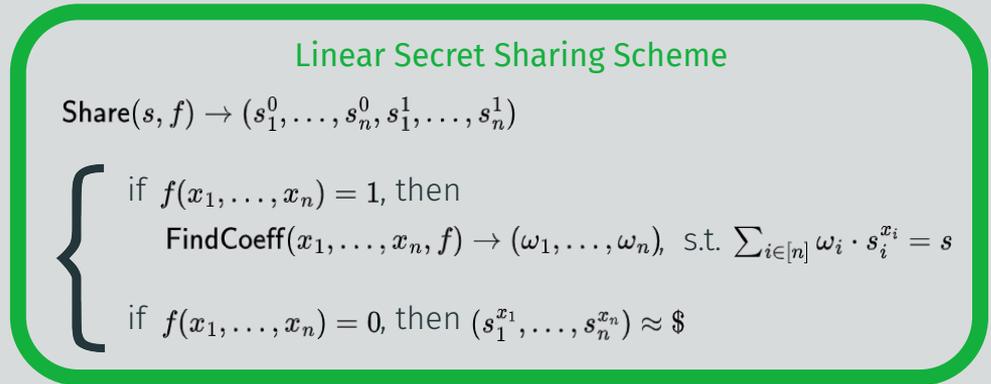
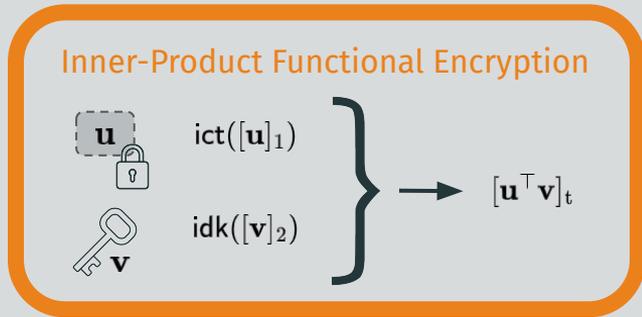
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[C:AYY22]	NC <sup>1</sup>	KOALA, LWE	✗	✗	2	✓
[C:ARYY23]	P	private-coin evasive LWE, tensor LWE, LWE	✗	✗	const	✓
[TCC:PS24]	NC <sup>0</sup> (and more)	SXDH	✓	✓	const	✓
[this work]	succinctly enumerable	MDDH	✓	✓	const	✓
[this work]	succinctly enumerable	public-coin evasive LWE, LWE	✓	✓	const	✓

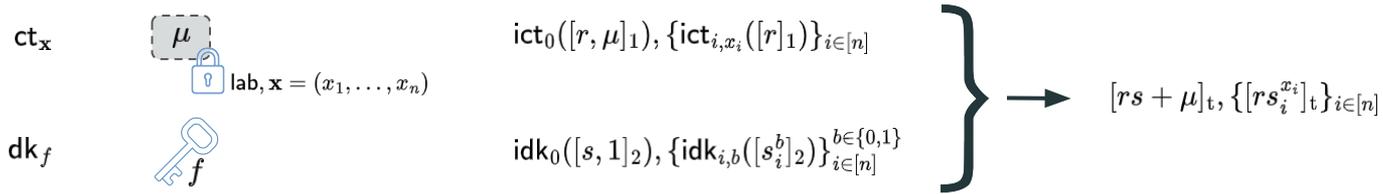
# Framework for Single-Client ABE



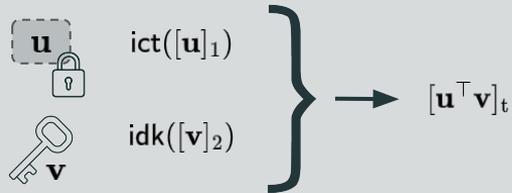
# Framework for Single-Client ABE



# Framework for Single-Client ABE



## Inner-Product Functional Encryption



## Linear Secret Sharing Scheme

$\text{Share}(s, f) \rightarrow (s_1^0, \dots, s_n^0, s_1^1, \dots, s_n^1)$

$\left\{ \begin{array}{l} \text{if } f(x_1, \dots, x_n) = 1, \text{ then} \\ \quad \text{FindCoeff}(x_1, \dots, x_n, f) \rightarrow (\omega_1, \dots, \omega_n), \text{ s.t. } \sum_{i \in [n]} \omega_i \cdot s_i^{x_i} = s \\ \text{if } f(x_1, \dots, x_n) = 0, \text{ then } (s_1^{x_1}, \dots, s_n^{x_n}) \approx \$ \end{array} \right.$

# Framework for Si

How to distribute this?

ct<sub>x</sub>

$\mu$



lab,  $\mathbf{x} = (x_1, \dots, x_n)$

dk<sub>f</sub>



f

$\text{ict}_0([r, \mu]_1), \{\text{ict}_{i, x_i}([r]_1)\}_{i \in [n]}$

$\text{idk}_0([s, 1]_2), \{\text{idk}_{i, b}([s_i^b]_2)\}_{i \in [n]}^{b \in \{0,1\}}$

$[rs + \mu]_t, \{[rs_i^{x_i}]_t\}_{i \in [n]}$

## Inner-Product Functional Encryption

$\mathbf{u}$

$\text{ict}([\mathbf{u}]_1)$

$\mathbf{v}$

$\text{idk}([\mathbf{v}]_2)$

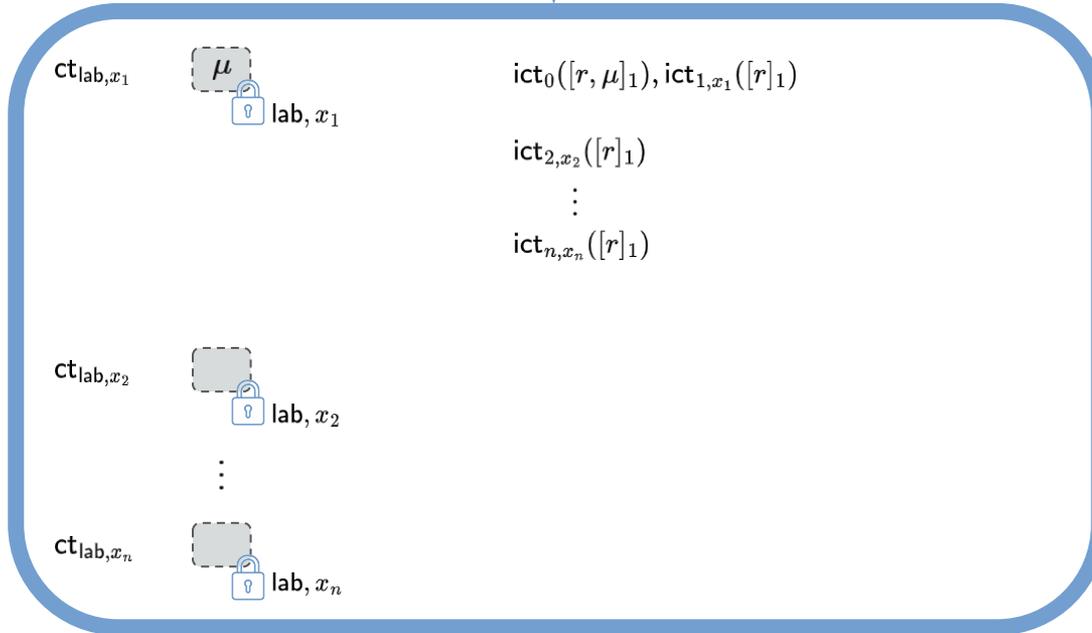
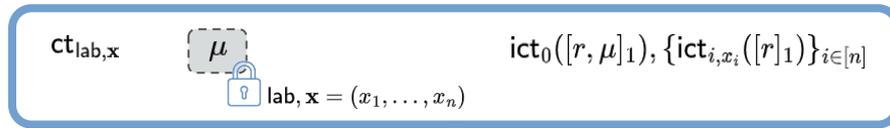
$[\mathbf{u}^\top \mathbf{v}]_t$

## Linear Secret Sharing Scheme

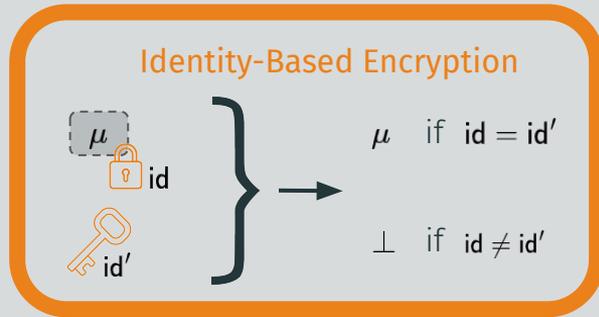
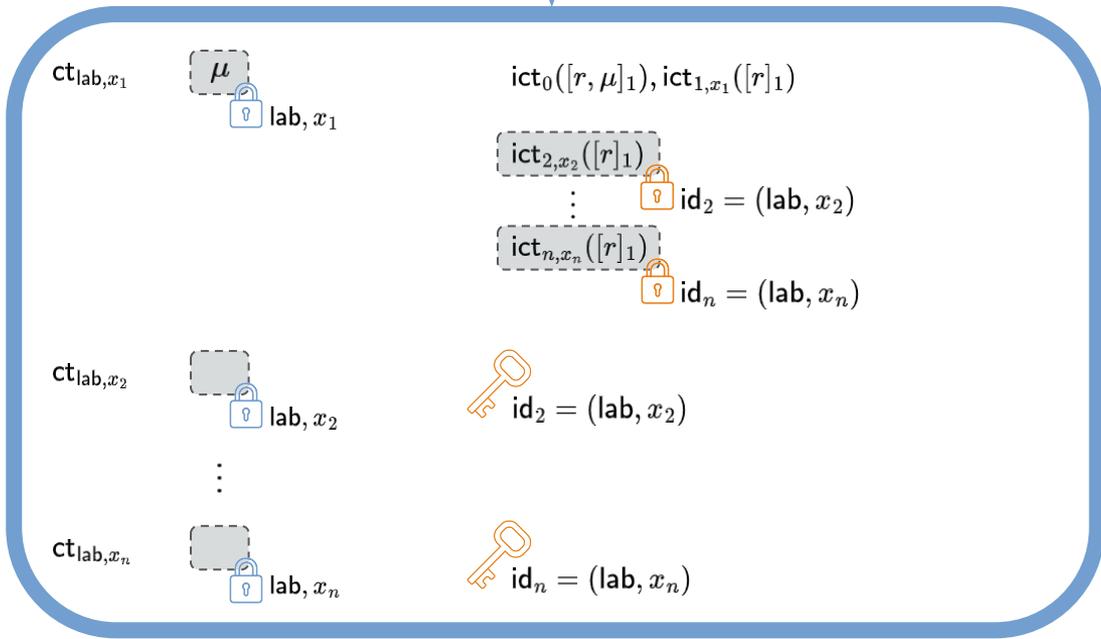
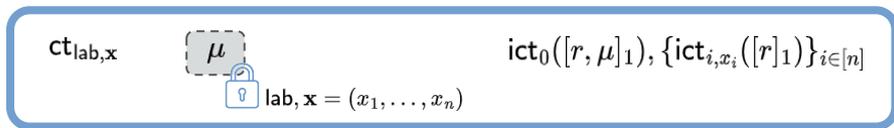
$\text{Share}(s, f) \rightarrow (s_1^0, \dots, s_n^0, s_1^1, \dots, s_n^1)$

$\left\{ \begin{array}{l} \text{if } f(x_1, \dots, x_n) = 1, \text{ then} \\ \quad \text{FindCoeff}(x_1, \dots, x_n, f) \rightarrow (\omega_1, \dots, \omega_n), \text{ s.t. } \sum_{i \in [n]} \omega_i \cdot s_i^{x_i} = s \\ \text{if } f(x_1, \dots, x_n) = 0, \text{ then } (s_1^{x_1}, \dots, s_n^{x_n}) \approx \$ \end{array} \right.$

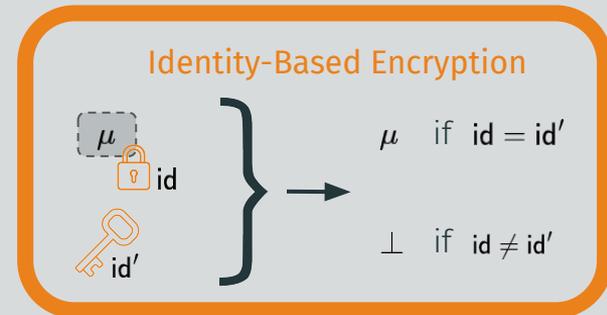
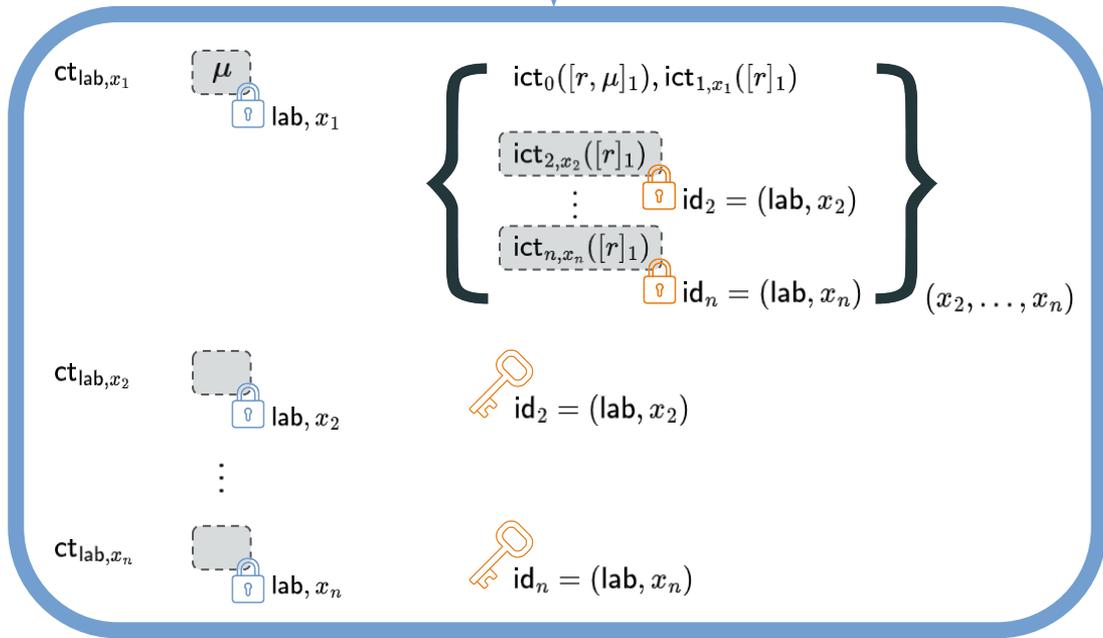
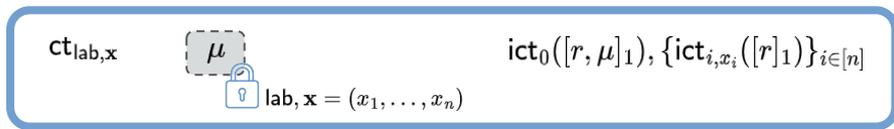
# Distributed Encryption



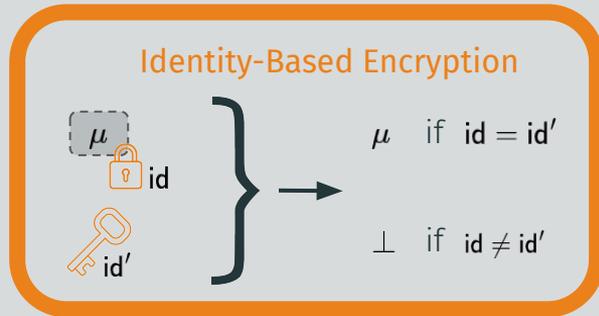
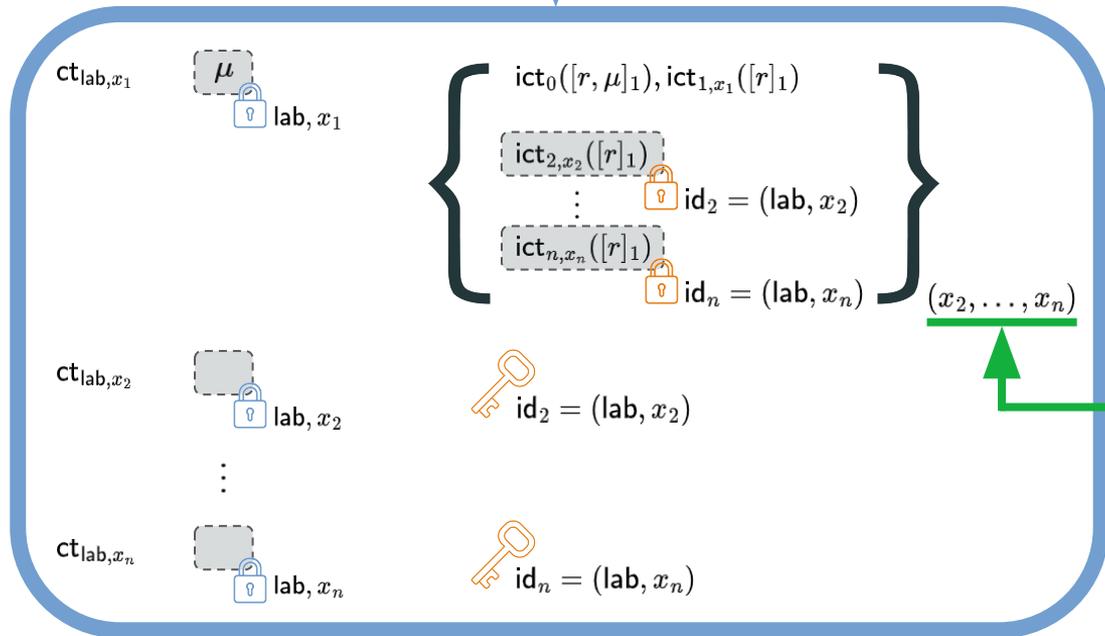
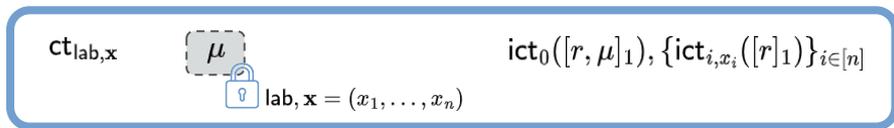
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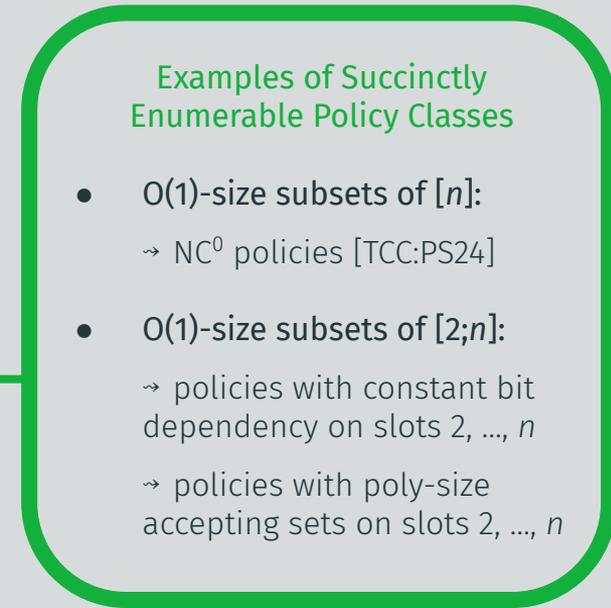
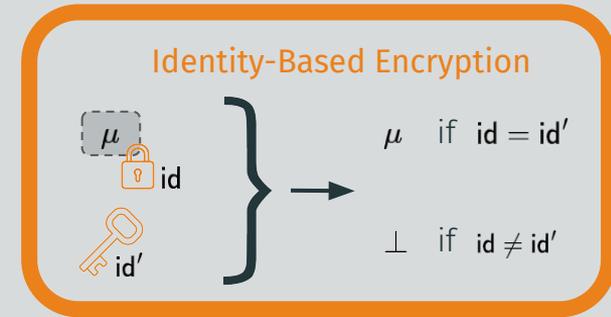
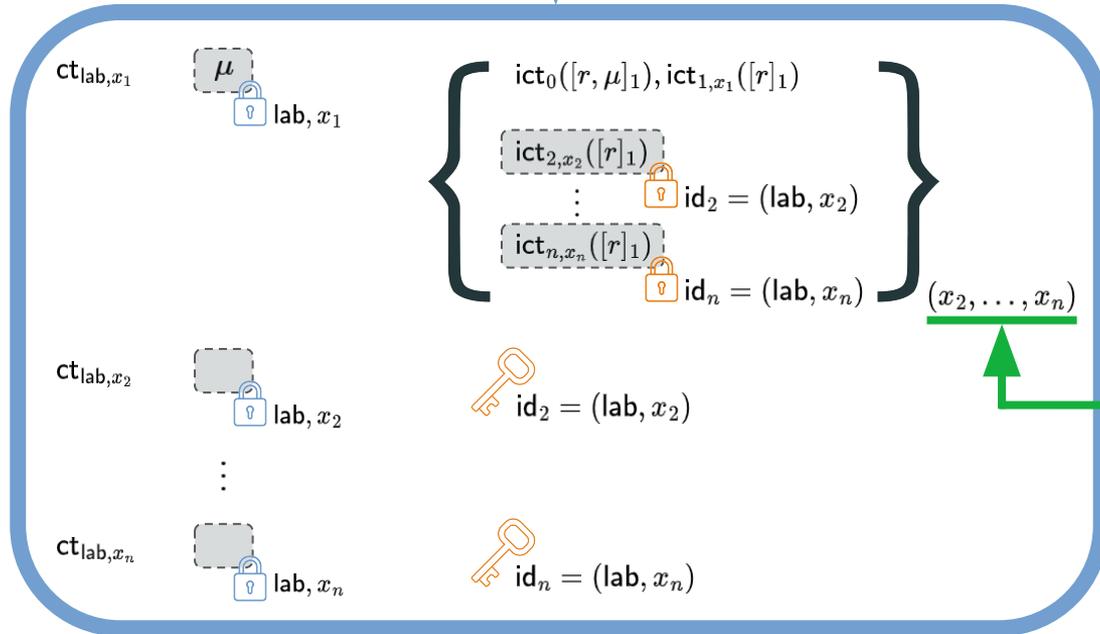
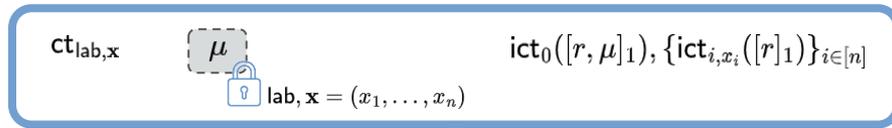
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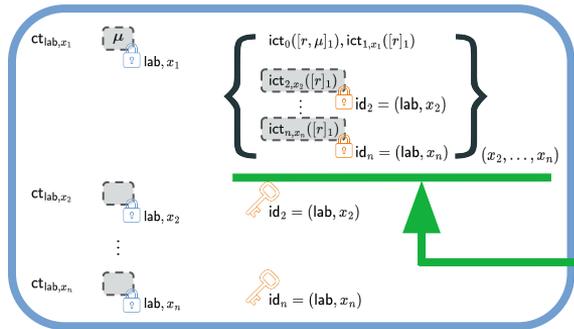
Which  $(x_2, \dots, x_n) \in \{0, 1\}^{n-1}$  do we need?

- $O(1)$ -size subsets of  $[n]$ :  
 $\rightarrow NC^0$  policies [TCC:PS24]
- $O(1)$ -size subsets of  $[2;n]$ :  
 $\rightarrow$  policies with constant bit dependency on slots  $2, \dots, n$   
 $\rightarrow$  policies with poly-size accepting sets on slots  $2, \dots, n$

# Distributed Encryption



# Succinct Enumerability

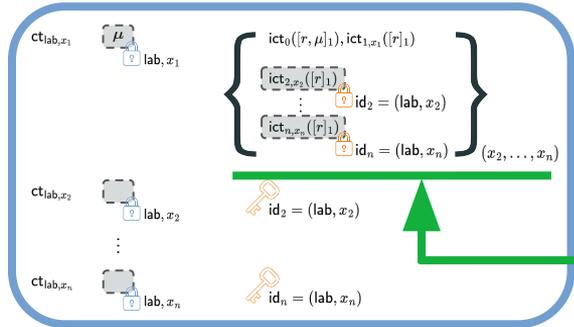


## Succinct Enumerability

there exists  $V \subseteq \{0, 1, \star\}^{[2; m]}$  such that

- succinctness:  $|V| = \text{poly}(\lambda)$

# Succinct Enumerability



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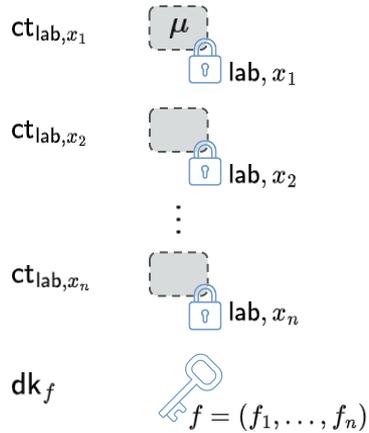
there exists  $V \subseteq \{0, 1, \star\}^{[2; n]}$  such that

- **succinctness:**  $|V| = \text{poly}(\lambda)$
- **correctness:** for all  $\mathbf{x}$  such that  $f(\mathbf{x}) = 1$ , there exists  $\mathbf{v} \in V$  such that  $\mathbf{x} \in X_{\mathbf{v}}$
- **security:**  $f$  is constant on  $X_{\mathbf{v}}$  for each  $\mathbf{v} \in V$

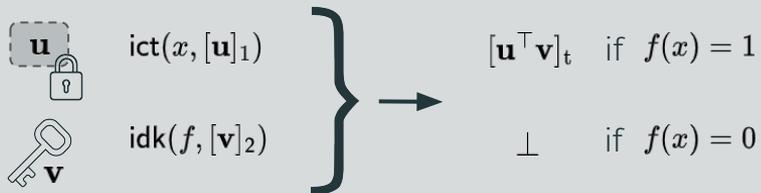
## Notation

$$X_{\mathbf{v}} = \{\mathbf{x} \in \{0, 1\}^n \mid \forall i \in [2; n] : v_i \neq \star \implies v_i = x_i\}, \quad \text{e.g., } X_{(1, \star, 1, 0, \star)} = \{(1, 0, 1, 0, 0), (1, 0, 1, 0, 1), (1, 1, 1, 0, 0), (1, 1, 1, 0, 1)\}$$

# Construction for Conjunction Policies



## Attribute-Based Inner-Product Functional Encryption



# Construction for Conjunction Policies

$ct_{lab, x_1}$	 $\mu$  lab, $x_1$	$ict_1(x_1, [t_{lab,1}, r]_1), c = [\mu]_t + H([r]_t)$
$ct_{lab, x_2}$	 $\dots$  lab, $x_2$	$ict_2(x_2, [t_{lab,2}]_1)$
$ct_{lab, x_n}$	 $\dots$  lab, $x_n$	$ict_n(x_n, [t_{lab,n}]_1)$
$dk_f$	 $f = (f_1, \dots, f_n)$	$idk_1(f_1, [s, 1]_2), \{idk_i(f_i, [s]_2)\}_{i \in [2;n]}$

## Attribute-Based Inner-Product Functional Encryption

 $\mathbf{u}$	$ict(x, [\mathbf{u}]_1)$	} →	$[\mathbf{u}^\top \mathbf{v}]_t$ if $f(x) = 1$
 $\mathbf{v}$	$idk(f, [\mathbf{v}]_2)$		$\perp$ if $f(x) = 0$

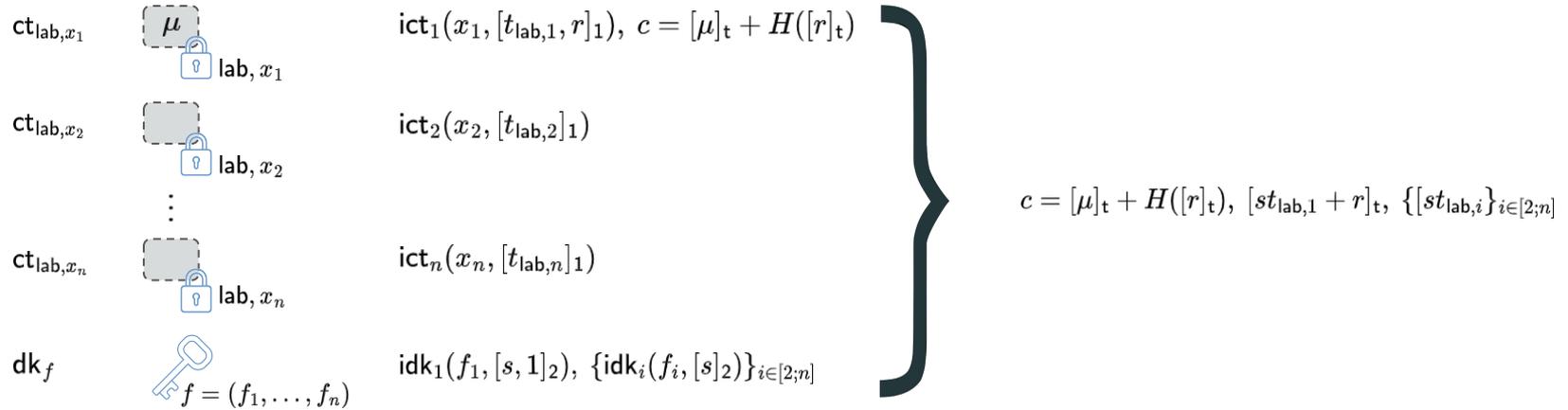
$$t_{lab,1} = - \sum_{i=2}^n \text{PRF}_{K_i}(\text{lab})$$

$$t_{lab,2} = \text{PRF}_{K_2}(\text{lab})$$

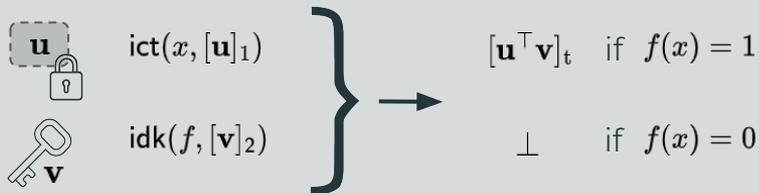
$$\vdots$$

$$t_{lab,n} = \text{PRF}_{K_n}(\text{lab})$$

# Construction for Conjunction Policies



## Attribute-Based Inner-Product Functional Encryption



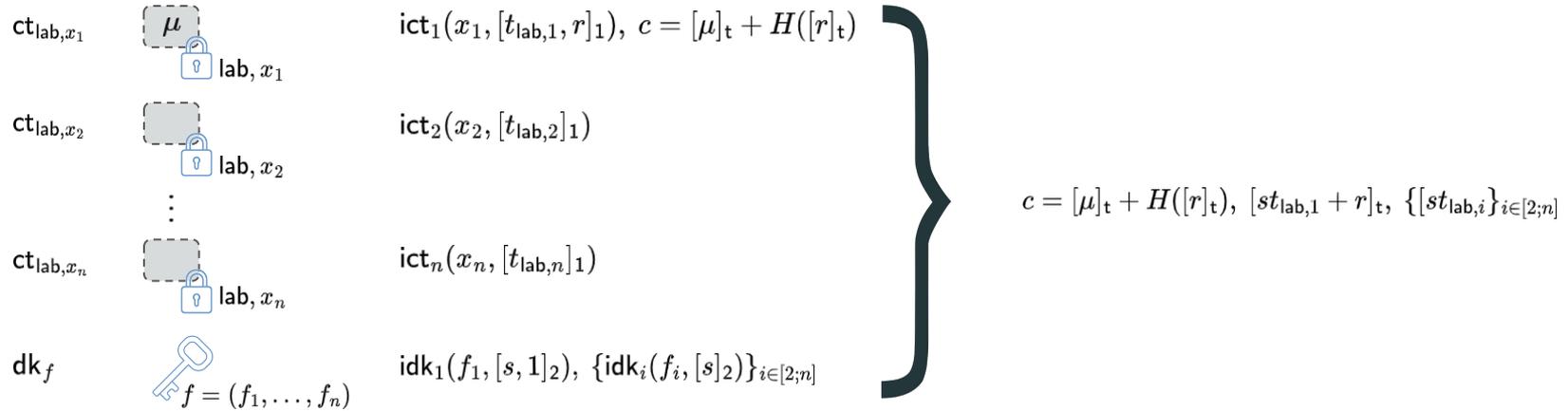
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$$\vdots$$

$$t_{lab,n} = \text{PRF}_{K_n}(\text{lab})$$

# Construction for Conjunction Policies

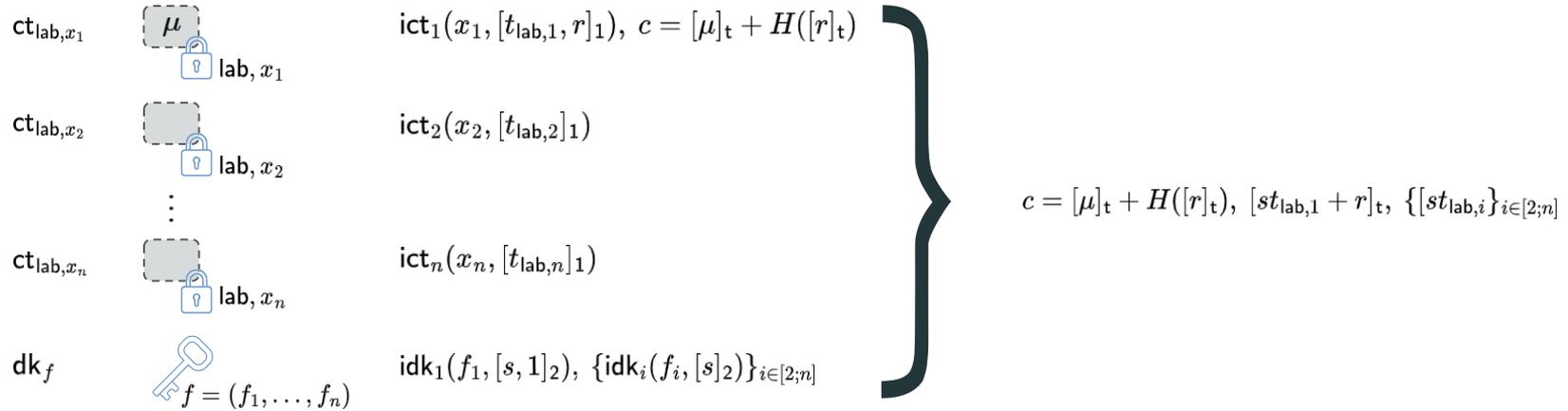


## Security

$f_1(x_1) = 0$ : 

$f_i(x_i) = 0$  for  $i \in [2;n]$ :

# Construction for Conjunction Policies



## Security

$f_1(x_1) = 0$ : 

$f_i(x_i) = 0$  for  $i \in [2;n]$ : apply DDH and PRF security, then argue  $\{[s'_{lab} + r_j]_t, H([r_j]_t)\}_j \approx_c \{[s'_{lab} + r_j]_t, [\delta_j]_t\}_j$  

# Conclusion

## Construction of MC-ABE for Conjunction Policies

- conjunctions of  $NC^1$  from MDDH on pairings
- conjunctions of P from LWE with bounded collusions

## Construction of MC-ABE for Global Policies

- succinctly enumerable policy classes
- instantiation from MDDH on pairings or public-coin evasive LWE

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Thank you for your attention!



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